

VU Research Portal

Passenger advice and rolling stock rescheduling under uncertainty for disruption management

van der Hurk, Evelien; Kroon, Leo; Maróti, Gábor

published in

Transportation Science
2018

DOI (link to publisher)

[10.1287/trsc.2017.0759](https://doi.org/10.1287/trsc.2017.0759)

document version

Publisher's PDF, also known as Version of record

document license

Article 25fa Dutch Copyright Act

[Link to publication in VU Research Portal](#)

citation for published version (APA)

van der Hurk, E., Kroon, L., & Maróti, G. (2018). Passenger advice and rolling stock rescheduling under uncertainty for disruption management. *Transportation Science*, 52(6), 1391-1411.
<https://doi.org/10.1287/trsc.2017.0759>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal ?

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

E-mail address:

vuresearchportal.ub@vu.nl



Transportation Science

Publication details, including instructions for authors and subscription information:
<http://pubsonline.informs.org>

Passenger Advice and Rolling Stock Rescheduling Under Uncertainty for Disruption Management

Evelien van der Hurk, Leo Kroon (deceased), Gábor Maróti

To cite this article:

Evelien van der Hurk, Leo Kroon (deceased), Gábor Maróti (2018) Passenger Advice and Rolling Stock Rescheduling Under Uncertainty for Disruption Management. Transportation Science 52(6):1391-1411. <https://doi.org/10.1287/trsc.2017.0759>

Full terms and conditions of use: <https://pubsonline.informs.org/Publications/Librarians-Portal/PubsOnLine-Terms-and-Conditions>

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2018, INFORMS

Please scroll down for article—it is on subsequent pages



With 12,500 members from nearly 90 countries, INFORMS is the largest international association of operations research (O.R.) and analytics professionals and students. INFORMS provides unique networking and learning opportunities for individual professionals, and organizations of all types and sizes, to better understand and use O.R. and analytics tools and methods to transform strategic visions and achieve better outcomes.

For more information on INFORMS, its publications, membership, or meetings visit <http://www.informs.org>



Passenger Advice and Rolling Stock Rescheduling Under Uncertainty for Disruption Management

Evelien van der Hurk,^a Leo Kroon,^{b,c,*} Gábor Maróti^{c,d}

^a DTU Management Engineering, Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark; ^b Rotterdam School of Management, Erasmus University Rotterdam, 3000 DR Rotterdam, Netherlands; ^c Netherlands Railways, 3511 ER Utrecht, Netherlands;

^d Vrije Universiteit Amsterdam, 1081 HV Amsterdam, Netherlands

*Deceased

Contact: evdh@dtu.dk,  <http://orcid.org/0000-0002-4521-3283> (EvdH); g.maroti@vu.nl,  <http://orcid.org/0000-0001-7715-5105> (GM)

Received: May 18, 2016

Revised: November 9, 2016; January 23, 2017

Accepted: January 30, 2017

Published Online in Articles in Advance:
October 9, 2018

<https://doi.org/10.1287/trsc.2017.0759>

Copyright: © 2018 INFORMS

Abstract. Operators and passengers need to adjust their plans in cases of large-scale disruptions in railway networks. Where most previous research has focused on the operators, this paper studies the combined support of both in a system where passengers have free route choice. In cases of disruption, passengers receive route advice, which they are not required to follow: passengers' route choice depends on the route advice and the timetable information available to them. Simultaneous to providing advice, rolling stock is rescheduled to accommodate the anticipated passenger demand. The duration of the disruption is uncertain, and passenger flows arise from a complex interaction between the passengers' route choices and the seat capacity allocated to the trains. We present an optimization-based algorithm that aims to minimize passenger inconvenience through provision of route advice and rolling stock rescheduling, where the advice optimization and rolling stock rescheduling modules are supported by a passenger simulation model. The algorithm aims to include and evaluate solutions under realistic passenger behavior assumptions. Our computational tests on realistic instances of Netherlands Railways indicate that the addition of the travel advice effectively improves the service quality to the passengers more than only rescheduling rolling stock, even when not all passengers follow the advice.

Funding: This work was supported by the Danish Innovationsfonden [Grant 4109-00005B, IPTOP] and the Netherlands Organisation for Scientific Research (NWO) [Grant 600.645.000.08, ComPuTr].

Supplemental Material: The online appendix is available at <https://doi.org/10.1287/trsc.2017.0759>.

Keywords: combinatorial optimization • disruption management • uncertainty • public transport • rolling stock rescheduling • passenger advice

1. Introduction

Urban public transportation systems around the world provide service to millions of passengers every day. Reliable service is essential to maintaining, and possibly increasing, the ridership of this efficient and sustainable mode of transportation. During major disruptions, it is imperative that operators have quick and effective disruption management strategies to diminish both the passenger delay and the long-term negative effects on ridership due to loss of confidence. This paper shows that the provision of targeted advice to passengers can significantly improve the service provided to passengers during major disruptions.

In cases of disruption, communication with passengers can happen almost instantly (e.g., using smartphones or tablets), while spare rolling stock may need hours to reach the locations with an elevated demand. Prompt route advice can immediately help passengers to avoid capacity bottlenecks that are impossible to prevent by rolling stock rescheduling measures alone. The capacity shortages depend on the available train capacity and the reaction of all passengers. These two aspects

are unknown to an individual passenger; therefore, the advice from the operator is essential. In systems with no seat reservations and free route choice, the provided advice needs to assist individual passengers' best interests; otherwise, the passenger will decide not to follow the advice. Thus, our problem setting is far more complex than integrated passenger (re)routing in systems with seat reservations, such as airlines.

In this research, we assume that the timetable has already been adjusted to the disrupted situation. That is, the departure and arrival times of the services are given, and no new services can be added, although services may be canceled. It is an interesting and challenging direction for future research to extend our algorithm to include timetabling decisions as well.

This paper addresses the integrated problem of passenger route advice and rolling stock rescheduling in systems with free route choice. Passengers receive advice in the form of a recommended route, compete for space, and react dynamically to changes in the timetable and available capacity. Therefore, the service quality

passengers experience results from a complex interaction between passengers and capacity. The objective is to minimize the passenger inconvenience in terms of a weighted sum of waiting time, in-vehicle time, and transfers, and to achieve this at a reasonable operating cost. To make our approach more realistic, we assume that the length of the disruption is uncertain and that passengers may disregard the advice.

The contribution of this paper is twofold. First, we propose a new mathematical optimization algorithm for the problem of combined passenger route advice and rolling stock rescheduling under more realistic passenger behavioral assumptions. This model aims to minimize passenger inconvenience in a system where passengers have free route choice. Second, we demonstrate through several case studies based on a realistic, large, and complex passenger rail network that the algorithm is viable and may yield significant benefits for disrupted passengers. Specifically, we find that the provision of advice yields benefits to the passengers even when the duration of the disruption is uncertain and not all passengers follow the advice. Moreover, good solutions can be found reasonably quickly.

The paper is organized as follows. Section 2 describes the overall model and solution process. Section 3 reviews the related literature. Section 4 describes the underlying passenger simulation model, the assumptions on passenger behavior, and the concept and function of advice. Section 5 presents the mathematical optimization model for advice optimization and discusses the rolling stock optimization, both for a deterministic disruption duration. Section 6 extends the models to handle disruptions with an uncertain duration. Finally, Section 7 is devoted to the results of our case study, and Section 8 discusses these results.

2. The Big Picture

Our algorithm is inspired by Kroon, Maróti, and Nielsen (2014), who solved the rolling stock rescheduling problem by iterating between a passenger simulation model feeding back information on the *demand per trip* to the rolling stock optimization model. The rolling stock optimization model in turn defines the *capacity per trip* for the passenger simulation. This paper differs in two important ways. First, an advice optimization component is added that supports passengers, for example, by warning them about capacity bottlenecks. As a result, changes are also made in the passenger simulation model and assumptions on passenger behavior. Second, the disruption length is considered to be uncertain, unlike the deterministic length assumed in Kroon, Maróti, and Nielsen (2014).

Our algorithm for the *advice and rolling stock rescheduling problem with uncertainty* (ARSRU) depicted in Figure 1 consists of two subalgorithms: an advice algorithm and a rolling stock rescheduling algorithm that is similar to Kroon, Maróti, and Nielsen (2014). The

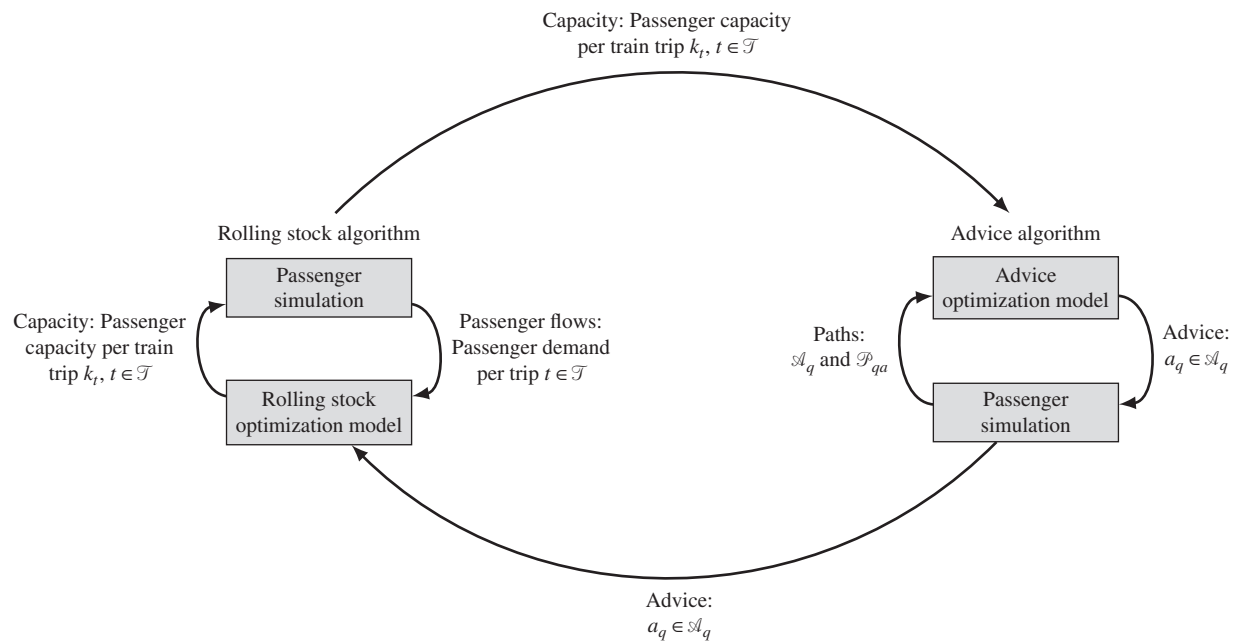
subalgorithms are iteratively solved with the objective to minimize expected total passenger inconvenience. The best advice depends on the available *capacity* per trip, defined by the rolling stock algorithm. The best rolling stock schedule depends on the demand per trip, which depends on the *advice*. We discuss here, as well as in Sections 4 and 5, the algorithm for a deterministic disruption duration; Section 6 explains the necessary changes for an uncertain disruption duration.

Both the advice and rolling stock algorithm contain two components: a mathematical optimization model and a passenger simulation model (Section 4). The same passenger simulation model is used in each component to model the interaction between passengers, the advice, and the rolling stock schedule. The passenger simulation depends on the provided *advice* path per passenger, defined in the advice optimization model (Section 5.1), and the available *capacity* per trip, defined in the rolling stock optimization model (Section 5.2). The passenger simulation model provides information to the advice optimization model on the *paths* of passengers, both realized paths and possible new recommended paths that could reduce passengers' delays. The passenger simulation model provides information on the *passenger flows*, in terms of the passenger demand per trip, to the rolling stock optimization model. The passenger simulation is our best model for the emerging passenger flows and therefore also serves as an evaluation tool to compute the passenger inconvenience in each of the subalgorithms under realistic passenger behavior assumptions, which includes passengers not following the advice.

Input to our model consists of a timetable, an adjusted timetable to the disruption, an initial rolling stock circulation, and the passenger demand defined as a set of passenger groups. Each passenger group represents a number of passengers planning to travel from an origin station to a destination station at a specific time. The algorithm is intended to support the operator at the start of a disruption. A solution defines a rolling stock circulation and an advice per affected passenger group. The *advice* is a path consisting of a specific set of station-to-station trips connecting the origin to the destination of the passenger.

The ARSRU algorithm starts with the rolling stock algorithm. This very first iteration is initialized by calling the passenger simulation with unlimited capacity, with all passengers following the shortest path in the timetable, after which the rolling stock rescheduling model is solved. Kroon, Maróti, and Nielsen (2014) found, in line with our results, that the rolling stock algorithm converges quickly with this initialization. The advice algorithm is run after the rolling stock algorithm, as its purpose is to reduce any passenger delay caused by capacity shortages that the rolling stock

Figure 1. ARSRU Algorithm Outline



schedule was not able to prevent (when all passengers follow the absolute shortest path).

3. Literature Review

Disruption management focuses on recovering operations after an unexpected event has made the original operational plans infeasible. Within public transport, this topic was first covered in airlines (Barnhart, Belobaba, and Odoni 2003, Kohl et al. 2007) and later railway transportation (Jespersen-Groth et al. 2009). The objective is commonly to minimize a weighted combination of operational costs and passenger inconvenience. Disruption management concerns the reaction to major disturbances—e.g., disturbance resulting from the closure of a track for several hours. Schöbel (2007) and Qu, Corman, and Lodewijks (2015) describe several models and provide a review for the separate field of delay management that considers smaller disturbances. Cacchiani et al. (2014) provide a general overview of disruption management in railways, while the short review below is limited to papers related to the current research.

Disruption Management for Airlines. Minimization of operating costs is one of the main objectives in airline disruption management (Barnhart, Belobaba, and Odoni 2003), to which, as a secondary objective, the minimization of passenger inconvenience is sometimes added. Lan, Clarke, and Barnhart (2006) indirectly minimize passenger delay by minimizing delays of individual aircraft. Bratu and Barnhart (2006) are one of the first to consider passenger *recovery* during disruptions in airlines. They include the passenger recovery in the decision of which planes to delay or cancel.

They propose two models: one where the delay costs for passengers for each decision are estimated outside the model, and one where passengers are reassigned explicitly. Although the latter has a more exact representation of passenger inconvenience, it is computationally much more expensive than the first, which is also successful in improving passenger service. Maher (2016) presents a model focused on point-to-point networks where passengers' alternative travel arrangements are uniquely linked to the choice to cancel a flight. Hu et al. (2016) present a GRASP-based algorithm that combines a greedy heuristic to construct aircraft rerouting solutions, with a local search heuristic. Their approach includes an operator controlled passenger assignment model. Hu et al. (2015) consider a reduced time-band network and an operator-controlled reassignment of passengers to flights with a later departure that have the same origin and destination as the canceled flight. Their objective is to minimize the total cost of recovering flights and the costs of rerouting passengers. However, railway operators do not have the power to assign passengers to new routes in passenger rail systems without seat reservations; therefore, these models cannot be directly applied in railways.

Trains and Rolling Stock. Fioole et al. (2006) describe a rolling stock scheduling for the setting of Netherlands Railways (NS), with a main focus on minimizing operational costs. The current paper's rolling stock model is an extension of Fioole et al. (2006). Cadarso and Marin (2011) consider rolling stock rescheduling in a rapid transit network for the Spanish operator RENFE. Passengers are considered as a fixed

demand between consecutive stations in given time intervals. The model includes limited timetabling decisions as well. Haahr et al. (2016) compare path- and composition-based models for rolling stock rescheduling and find that both models could be fast enough for real-time support. Furthermore, Haahr, Pisinger, and Larsen (2015) propose a fast branch-and-price model for rolling stock rescheduling, as well as a model for integrating rolling stock rescheduling with depot planning. Samá et al. (2016) propose a scheduling and routing metaheuristics for train scheduling for traffic management in railway networks that minimizes the maximum consecutive delay. Their algorithm combines several neighborhood search schemes to find new schedules fast for busy rail networks during significant disturbances. In a comparison test, their algorithm outperforms a tabu-search and a MILP formulation solved with a commercial solver. Dauzère-Pérés et al. (2015) propose a Lagrangian heuristic for the integrated scheduling of crew and rolling stock, by solving the two subproblems for rolling stock scheduling and crew rescheduling including a few coupling constraints using a Lagrangian relaxation scheme. None of these approaches are focused explicitly on reducing passenger inconvenience including dynamic passenger flows.

Uncertainty and Robustness. Cacchiani et al. (2012) propose a two-stage optimization model using Benders' decomposition to solve the robust rolling stock scheduling problem for finding better rolling stock plans, subject to a fixed passenger demand. Nielsen, Kroon, and Maróti (2012) extend the model of Fioole et al. (2006) to a rolling horizon approach for rolling stock rescheduling. Veelenturf et al. (2016) propose *quasi-robust* scheduling for dealing with an uncertain disruption duration in the context of crew rescheduling. The aim is to compute crew duties for the optimistic scenario (i.e., shortest disruption duration) in such a way that they can be recovered even if another scenario takes place.

Passengers and Information. Cadarso, Marín, and Maróti (2013) present a model for rolling stock and (limited) timetable rescheduling where passenger inconvenience is minimized based on a dynamic assignment of demand in the resulting schedule. The passenger demand is iteratively updated outside of the optimization model. Kroon, Maróti, and Nielsen (2014) combine a rolling stock rescheduling model with passenger simulations in an iterative framework to balance passenger inconvenience and operational costs. The current paper extends the work of Kroon, Maróti, and Nielsen (2014). Veelenturf et al. (2017) describe an extension of Kroon, Maróti, and Nielsen (2014) for minimizing passenger inconvenience by also allowing minor changes to the

timetable. Parbo, Nielsen, and Prato (2014) propose a genetic-algorithm to reduce passenger waiting times by changing the departure times of buses; the solutions are evaluated using a detailed passenger assignment model. Cadarso, Maróti, and Marín (2015) consider integrated rolling stock rescheduling and timetabling for disruptions in railways, with the main focus on providing pragmatic plans by limiting the recovery period and the number of schedule changes. Similar to Cadarso, Marín, and Maróti (2013), passenger flows are dynamic and passengers update their path in reaction to a disruption. In all five papers, passengers update their path based on the timetable and do not receive advice from the operator helping them to avoid capacity shortages.

Delay management applications consider passenger inconvenience, as well. Sato, Tamura, and Tomii (2013) minimize passenger delays in timetable rescheduling. Dollevoet et al. (2012) consider delay management with dynamic rerouting of passengers: the passenger flows are not fixed, but depend on the decisions of letting trains wait for a feeder train or not. Corman et al. (2017) include both macroscopic decisions on delay management and dynamic passenger routing in a microscopic timetabling model for small delays. We refer to Schöbel (2007) and Qu, Corman, and Lodewijks (2015) for a more extensive review.

Koutsopoulos et al. (2011) demonstrate that passengers can profit from having real-time information on the current state of the timetable, using the mesoscopic simulation model BusMezzo that explicitly models the interaction between passengers and the public transport system. Watkins et al. (2011) show that access to real-time information of the schedule can reduce passengers' waiting time and increase their satisfaction with the system, by conducting a real-life experiment with the OneBusAway transit traveler information application.

Although, in general, the attention to passengers is increasing in the field of public transport, few papers focus on modeling dynamic passenger flows. The extensive review of Parbo, Nielsen, and Prato (2015) on passenger-related timetabling contains very few models with dynamic passenger demand modeling, and none is similar to the context of passenger behavior considered in this paper. Therefore, we believe the current paper, with its aim to model more realistic passenger behavior in the context of an optimization model for railway planning *where passengers have free route choice*, forms a novel contribution to the body of public transport research.

4. Passenger Simulation, Behavior, and Advice

An important aspect of this paper is the modeling of more realistic passenger behavior than an operator

control or flow cost minimization model for systems with free route choice. The passenger simulation model serves as our best model for passenger behavior. To a limited extent, this behavior is also included in the advice optimization model. The three key characteristics of the modeled passenger behavior are as follows.

- Passengers act out of *self-interest*: altruistic behavior to benefit the group does not occur.
- Passengers *compete* with each other for space in case of insufficient capacity: passengers already on board of a train keep their seats, while boarding passengers compete with each other for the remaining space.
- Passenger behavior is *dynamic*: they adapt their route when trips are canceled or they are unable to board a train.

The following concepts are used in this section and throughout this paper:

A *timetable* is a set of trips, where a *trip* t is a train ride between two consecutive stops at a specific time. A disruption, in the context of this paper, causes the cancelation of a set of trips. We denote by \mathcal{T} the original or planned timetable, and by \mathcal{T}^* the timetable adjusted to the disruption.

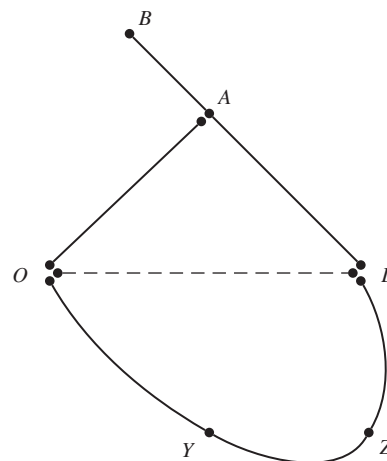
A passenger's *path* is defined as an ordered set of trips in the timetable. These paths correspond to directed paths in the time-space graph representation $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of the timetable. It contains a node for each station and for each time instant when a train departs from, or arrives at, that station. The set $\mathcal{E}_{\text{trip}}$ contains an arc for every trip $t \in \mathcal{T}$, the set $\mathcal{E}_{\text{wait}}$ contains arcs connecting nodes of the same station over time, and we define $\mathcal{E} := \mathcal{E}_{\text{trip}} \cup \mathcal{E}_{\text{wait}}$. Thus, arcs in $\mathcal{E}_{\text{trip}}$ represent passengers traveling by train, and arcs in $\mathcal{E}_{\text{wait}}$ represent passengers waiting at a station. A straightforward modification of this intuitive graph allows us to account for transfers as well. This representation of the graph is used to find earliest-arriving paths using a standard minimum cost path search.

A *passenger group* $q \in \mathcal{Q}$ represents a number of passengers with the same origin station, destination station, departure time, and planned path in the original timetable \mathcal{T} .

Section 4.1 illustrates the complex interaction between passengers and capacity with a small example. Section 4.2 describes the passenger simulation model and assumptions on passenger behavior. Finally, Section 4.3 defines the advice and describes how possible options for advice are generated.

These topics are discussed in the context of a known disruption duration; extensions to an uncertain disruption duration are presented in Section 6. An overview of all notation used in the paper is provided in Table 1.

Figure 2. Public Transport Network



4.1. Example: Complex Interaction Between Passengers and Capacity

The complex interaction between passengers and capacity is explained using the example network in Figure 2. Because of a track blockage, the direct train from O to D is canceled. Passengers can choose between two detours: the faster route O–A–D with transfer at A or the slower direct route O–Y–Z–D. The slower O–Y–Z–D train departs before the O–A–D train and is the fastest route if passengers are unable to board either the O–A train or the connecting A–D train due to capacity shortages.

Raising the capacity of the O–A train will only benefit detouring passengers if the connecting A–D train provides sufficient space for them. Otherwise, the elevated capacity of the O–A train creates a capacity bottleneck at station A as more passengers arrive than can leave. This illustrates the first interaction: the benefit to passengers of extending the capacity of one train depends on the capacity of other trains.

The passenger inconvenience can also be affected by the route choice of other passengers. For example, the (B–)A–D train might offer insufficient space to passengers at A when many B–D passengers have boarded the train at previous station B. All boarding passengers at A will have to compete with each other for the remaining seats. Passengers originating at A compete with passengers detouring through A from O for the remaining seats; thus, they may suffer a delay if the (B–)A–D train has an insufficient capacity.

Adjusting the train capacity alone is not always sufficient to prevent capacity bottlenecks: passengers also need to be aware of capacity shortages and alternative routes. One may reduce the capacity of the O–A train to the number of passengers able to board the A–D train. However, this step alone does not encourage the use of the O–Y–Z–D route: when the O–Y–Z–D train departs, the passengers are not yet aware of the capacity shortages on the O–A–D route. That is, shortening

Table 1. Notation and Terminology

Symbol	Explanation
\mathcal{T}	Timetable
\mathcal{D}	Set of disruptions with length $\delta \in \mathcal{D}$
\mathcal{E}	Set of arcs of timetable graph
\mathcal{Q}	Set of passenger-groups
\mathcal{A}_q	Set of recommended paths for passenger group q
\mathcal{P}_{qa}	Set of realized paths for passenger group q and advice a
c_p	Cost of realized path p
k_t	Capacity of trip $t \in \mathcal{T}$
x_{qpa}	Number of passengers of passenger-group q on realized path p for advice a
\mathcal{T}^δ	Timetable adjusted to disruption δ
ϕ	Probability of accepting the advice
\mathcal{V}	Set of nodes of timetable graph
w_q	Passengers in group $q \in \mathcal{Q}$
\mathcal{P}_{qat}	Set of realized paths for passenger group q and advice a traversing trip t
\mathcal{P}_{qa}^δ	Set of realized paths for passenger group q and advice a and disruption δ
k_t^δ	Capacity of trip $t \in \mathcal{T}$ in disruption δ
y_{qa}	Decision to select (1) or not (0) path a as advice to passenger-group q

the O – A train will only move the capacity bottleneck from A to O . By contrast, the suggestion to use the O – Y – Z – D route (because of capacities on the alternative O – A – D route) could convince passengers to use this route and thereby reduce their delay.

The provision of advice can help prevent the occurrence of capacity bottlenecks and reduce passenger inconvenience in a way that rolling stock rescheduling alone cannot. When an unavoidable capacity bottleneck is present at A , passengers at O will be better off when they receive, and follow, the advice to take the “slower” O – Y – Z – D route rather than compete, and lose this competition for space at A . Apart from reducing their own delay, following the O – Y – Z – D route may also reduce the delay of other passengers as the total demand on the A – D route declines, and even prevent capacity bottlenecks from occurring. A leading principle in this paper is that an advice route, such as the O – Y – Z – D route, must have immediate benefits for the passengers following this route. A crucial feature is that an operator cannot assign passengers to paths or reserve space for them, as all passengers have free route choice. Thus, the provided advice must be in the best interest of the passenger, as in the example. Moreover, we will evaluate solutions under different passenger behavioral assumptions, including several where not all passengers follow the advice.

4.2. Passenger Simulation and Behavior

The passenger simulation is our best model of passenger behavior. It serves as part of the *ARSRU* algorithm, and is used to evaluate the solution quality

of *ARSRU* solutions under different passenger behavioral assumptions regarding the acceptance of advice. This set-up was inspired by Kroon, Maróti, and Nielsen (2014).

The passenger behavior is described in Section 4.2.1. Section 4.2.2 provides a general outline of the passenger simulation model.

4.2.1. Passenger Behavior. The following three paths are used to describe and discuss passenger behavior throughout this research.

- The *planned* path is the passenger’s preferred path in case of no disruptions.
- The *recommended* path is the path that is advised to the passengers at the start of the disruption.
- The *realized* path is the passenger’s traveled path.

Intrinsic passenger behavior defines the planned path, including a reaction to the recommended path; the behavioral and interaction rules lead to the realized path. As mentioned above, a passenger group $q \in \mathcal{Q}$ represents a number of passengers with the same origin station, destination station, departure time, and planned path. All members of a passenger group have the same planned path, receive the same recommended path (advice), but may have different realized paths. Initial passenger groups may be split to model competition for seats and different reactions to the advice. For the sake of convenience, passenger groups can have a fractional size.

The passenger simulation allows a wide range of behavioral rules. In this paper, the following rules are implemented:

- A passenger’s planned path is the path with the earliest arrival time in the original timetable given the planned departure time of the passenger.
- A passenger accepts the recommended path of the advice with probability ϕ , otherwise the passenger follows the path with the earliest arrival time in the adjusted timetable. If $0 < \phi < 1$, the passenger group is split in two, with fraction ϕ following the advice and $1 - \phi$ following the earliest-arriving path in the adjusted timetable.
- When attempting to board a train, the passenger groups compete for the available capacity.
- When passengers are unable to board a train due to capacity shortages, passengers update their path to the path with the earliest-arriving path in the adjusted timetable.
- When anticipated delay is above a threshold, passengers stop their journey (e.g., chose an alternative mode, or not travel), and the operator incurs an additional penalty.

Although a wide range of different rules could be selected and implemented in the simulation, this set was chosen as it is the closest to the behavioral rules in Kroon, Maróti, and Nielsen (2014). That is, if $\phi = 0$, no passengers would follow the advice, and the resulting

behavior of our model is equivalent to the assumptions in Kroon, Maróti, and Nielsen (2014).

The passenger simulation allows for various ways of calculating the value of ϕ —e.g., it may depend on the relative quality of the recommended path in comparison to the earliest-arriving path in the adjusted timetable. The sensitivity analysis for different calculations of ϕ in the case study in Section 7 demonstrates that the ARSRU algorithm finds rolling stock schedules and advice that together reduces passenger inconvenience further than only rescheduling rolling stock in an approach similar to Kroon, Maróti, and Nielsen (2014), even when in the evaluation of the ARSRU solution not all passengers follow the advice.

4.2.2. Passenger Simulation. Our passenger simulation is a slightly altered version of the passenger simulation in Kroon, Maróti, and Nielsen (2014), to which we have added the concept of advice, different behavioral reactions to advice, and the uncertain disruption duration (which is explained in Section 6). The main concepts and assumptions important for the understanding of this paper are described here; technical details can be found in Kroon, Maróti, and Nielsen (2014).

Passenger Inconvenience. Passenger inconvenience is calculated as the weighted sum of the differences between the passengers' planned paths and the realized paths, as computed in the simulation. Paths are compared in terms of waiting time, in-vehicle time, and number of transfers. In our application, the passenger inconvenience is expressed as time difference or delay. However it is an easy extension to incorporate specific weights for waiting time or transfer time.

Input and Output. Input to the simulation consists of an initial timetable, a timetable adjusted to the disruption, a set of passenger groups \mathcal{Q} , a recommended path per passenger group a_q , $q \in \mathcal{Q}$, and the capacity per trip in the timetable defined by the rolling stock schedule. Output of the simulation is a set of realized paths and the number of passengers who have taken these paths. This defines both the passenger inconvenience and the demand per trip, the latter of which is input to the rolling stock optimization model. Moreover, the simulation algorithm provides realized and candidate recommended paths to the advice optimization module, as described in Section 4.3.

Algorithm. The passenger flows resulting from interaction with other passengers and the available capacity are simulated as follows.

- Passengers follow their planned path.
- Passengers adapt their path according to their behavior (Section 4.2.1):
 - at the start of the disruption τ_{start} ;
 - when they are unable to board a train.

- Passengers compete for space. Passengers already on board the train keep their seats, while boarding passengers compete with each other. If the capacity is insufficient, the passenger groups receive a portion of the capacity proportional to their size. Then, the groups are split: one portion can board the train while the other portion stays behind on the platform. Passengers unable to board replan as defined by their behavioral rules (e.g., to an earliest-arriving path in the timetable).

- Passengers break off their journey if the expected arrival time is after their *deadline*.

The concept of a deadline is included to limit computation time. It also allows us to include passengers choosing a different mode of transportation or deciding not to travel at all, and to penalize long delays more. The concepts of competition and deadline are inspired by the passenger simulation of Kroon, Maróti, and Nielsen (2014).

4.3. Advice

The passenger simulation generates both the set of realized paths and the set of candidate recommended paths; the advice optimization model will assign each passenger group to one recommended path and associated realized paths. The use of these paths helps incorporate elements of the passengers' behavior into the advice optimization model: passengers will receive recommended paths that are likely to be acceptable for them. For example, the realized paths never force passengers to disembark their direct train; the candidate recommended paths are generated such that these paths will (often) be in the best interest of the passenger receiving the advice. Ceder and Wilson (1986) proposed to precalculate paths to include behavioral constraints for a line planning model. For similar reasons, van der Hurk et al. (2016) pregenerated passenger paths for the multicommodity flow component of a shuttle planning problem for maintenance closures and found that this procedure led to an estimation of passenger inconvenience that is comparable to that of more realistic passenger-route choice models.

In our framework, the set of paths is iteratively extended in the passenger simulation. The iterative extension of a path-set within optimization is generally done using a pricing model. However, we are not aware of a pricing model that captures the complex behavioral constraints in our model (e.g., no passengers disembarking their direct train), and that can also express the complicated relationship between realized paths and recommended paths.

Section 4.3.1 defines the set of *attractive* paths that are considered as candidate recommended paths. Section 4.3.2 describes how these attractive paths are computed in the passenger simulation model, as well as the relationship between attractive paths and realized paths.

4.3.1. Advice and Attractive Paths. The main motivation for providing advice is to assist passengers in choosing the best paths for them during a disruption. In this paper, the set of candidate recommended paths is restricted to *attractive* paths of two types.

1. Earliest-arriving paths in the timetable graph \mathcal{G} independent of the available capacity, given the departure time of the passenger.
2. Paths in the timetable graph \mathcal{G} along which a passenger arrives earlier than on a path of the first type by *avoiding capacity shortages*.

In the example in Section 4.1, $O-A-D$ is of the first type and $O-Y-Z-D$ is of the second type. The difficulty is that paths of this second type depend on the provided capacity per trip and the route choice of other passengers—both of which will be altered in the solution process. Therefore, the use of paths of the second type should be considered as a heuristic approach.

At the start of the disruption, only the *affected* passengers receive advice in the form of a recommended path. Affected passengers are those whose planned path contains at least one of the disrupted trips, as well as those passengers who might be unable to board their train due to capacity shortages on their path. Moreover, one advice is selected per passenger group. That is, the advice is not fully personalized. This practical constraint allows for an easy integration with a journey planner application and ensures credibility of the advice, which could otherwise suffer from two travel companions receiving different recommended paths as advice. However, providing different advice to passengers of the same group is a straightforward extension of the advice optimization model that will be presented in Section 5.

4.3.2. Generation of Recommended Paths and Realized Paths. In the course of the ARSRU algorithm for each passenger group $q \in \mathcal{Q}$, a recommended path set \mathcal{A}_q is maintained. Moreover, a realized path set \mathcal{P}_{qa} is maintained for each passenger group $q \in \mathcal{Q}$ and each candidate recommended path $a \in \mathcal{A}_q$. New paths can be added to \mathcal{A}_q and \mathcal{P}_{qa} in *every* run of the simulation model.

The set \mathcal{A}_q is initialized with the earliest-arriving path in the timetable graph independent of the available capacity for each passenger group q . In each passenger simulation run, the set \mathcal{A}_q is extended by attractive paths of the second type—that is, new paths that, by avoiding capacity bottlenecks, are expected to have passengers arrive earlier than their current recommended path. New paths to \mathcal{A}_q are added when any of the passengers of group q are unable to board a train due to capacity shortages. The simulation checks, by way of a shortest path computation, if there exists a path with an earlier expected arrival time than the current anticipated path for passengers unable to board

the train, which leaves from their planned origin at the planned departure time and avoids any overcrowded trips—without anticipating the start of the disruption. If so, this path is added to \mathcal{A}_q .

This way of generating recommended paths ensures that only passengers affected by the disruption receive the advice. Passengers not affected by the disruption will receive the “advice” to follow the earliest-arriving path in the timetable independent of capacity shortages—which is equal to their planned path. Only when passengers are affected by the disruption (the earliest-arriving path in the original and the adjusted timetable differ), or face capacity shortages on their own paths that make them unable to board their preferred train, do they possibly receive advice containing a recommended path different from their planned path.

Two initial realized paths in \mathcal{P}_{qa} are constructed for each recommended path $a \in \mathcal{A}_q$: (1) where passengers follow the advice and (2) where passengers decide to not travel by train. This second path connects the passengers origin to their destination without the use of trains at a high (passenger inconvenience) cost and thus represents passengers leaving the train system. As no data is available on when and to what extent passenger demand declines, this feature is included foremost to assure the existence of a feasible solution, to penalize very long delays for individual passengers, and for computational reasons. However, the model is flexible enough to include passenger group-specific and delay-dependent decline of demand, if such information were available.

In every passenger simulation run, the set \mathcal{P}_{qa} is extended by the realized paths computed in the simulation of passengers in group q who received advice a . When a portion of the passengers are unable to board the train, multiple realized paths for one passenger group and advice will result from a single simulation run: namely, one for the passengers of the group able to board, and one for those passengers of the group that were not able to board that train. The realized path set \mathcal{P}_{qa} thus reflects more realistic passenger behavior in the case of capacity shortages and, for example, will not include paths where passengers disembark their direct train.

Capacity shortages arise depending on the paths (and thus advice) of other passenger groups, and on the capacity per trip defined by the rolling stock schedule. Therefore, the same advice could lead to different realized paths in passenger simulation runs with a change in rolling stock schedule, or a change in the advice provided to *other* passengers. Consequently, sets \mathcal{A}_q and \mathcal{P}_{qa} are continuously extended through the run of the ARSRU algorithm.

5. Mathematical Optimization Models for the Advice and Rolling Stock Rescheduling Algorithm

Section 5.1 presents the advice optimization model, and Section 5.2 presents the rolling stock rescheduling model. Within this section, the models are introduced in the context of a deterministic disruption duration; Section 6 will propose the extensions to incorporate the uncertain disruption duration.

5.1. Advice Optimization Model

The advice optimization supports the operator at the start of a disruption in selecting advice (Section 4.3) in the form of a single recommended path per affected passenger group $q \in \mathcal{Q}$, where a passenger group q represents w_q passengers departing at the same time, traveling from the same origin station to the same destination station. The objective is to minimize total passenger inconvenience caused by the disruption given capacity constraints defined by the rolling stock schedule. The model is path based, foremost to include some passenger behavioral constraints as motivated in Section 4.3, which also describes the construction of (attractive) recommended paths \mathcal{A}_q and realized paths \mathcal{P}_{qa} .

Input to the advice optimization model consists of a set of passenger groups \mathcal{Q} , a set of precomputed (attractive) recommended paths \mathcal{A}_q per passenger group $q \in \mathcal{Q}$, and a set of realized paths \mathcal{P}_{qa} for each recommended path $a \in \mathcal{A}_q$ and passenger group $q \in \mathcal{Q}$. A rolling stock schedule defines the capacity k_t for each trip t in the given timetable \mathcal{T}^δ that has been adjusted to the disruption. Furthermore, \mathcal{P}_{qat} represents the set of realized paths containing trip t for advice a , and c_{qp} captures the inconvenience of a single passenger of group q when traveling on realized path p . The costs c_{qp} depend solely on the costs of the arcs in the path that represent in-vehicle time, waiting time, and transfers. The limited available capacity is taken into account in the constraints.

Two types of decision variables are included in the model:

- y_{qa} , a binary variable indicating the selection of recommended path a as advice for passenger group q , $a \in \mathcal{A}_q$.
- x_{qpa} , a continuous variable indicating the number of passengers of passenger group q following realized path p belonging to advice a , $p \in \mathcal{P}_{qa}$, $a \in \mathcal{A}_q$.

The model is formulated as follows:

$$\min \sum_{q \in \mathcal{Q}} \sum_{a \in \mathcal{A}_q} \sum_{p \in \mathcal{P}_{qa}} c_{qp} x_{qpa}$$

subject to

$$\sum_{a \in \mathcal{A}_q} y_{qa} = 1 \quad \forall q \in \mathcal{Q}, \quad (1)$$

$$\sum_{p \in \mathcal{P}_{qa}} x_{qpa} = y_{qa} w_q \quad \forall q \in \mathcal{Q}, a \in \mathcal{A}_q, \quad (2)$$

$$\sum_{q \in \mathcal{Q}} \sum_{a \in \mathcal{A}_q} \sum_{p \in \mathcal{P}_{qat}} x_{qpa} \leq k_t \quad \forall t \in \mathcal{T}, \quad (3)$$

$$x_{qpa} \geq 0, y_{qa} \in \{0, 1\} \quad \forall q \in \mathcal{Q}, \forall a \in \mathcal{A}_q, \forall p \in \mathcal{P}_{qa}. \quad (4)$$

Objective. The objective function minimizes the overall delay of the passengers. Note that the operational costs do not change due to the selected advice, as the rolling stock schedule within the advice optimization module is fixed.

Constraints. Constraint (1) specifies that a single recommended path is selected as advice for each passenger group. Constraint (2) ensures that all passengers of group $q \in \mathcal{Q}$ are assigned to a realized path associated with the selected recommended path of the advice y_{qa} . Constraint (3) further restricts the assignment to not exceed the available capacity per trip.

Minimizing the total cost of this flow is not necessarily equivalent to minimizing each individual passenger's inconvenience independently. As the inconvenience of a specific route choice of one passenger depends on the route choice of other passengers, as explained in Section 4.1, the advice optimization cannot be solved for each passenger independently. Therefore, in model (1)–(4), the best advice for each passenger group is selected simultaneously, and the inconvenience is based on the expected resulting realized routes from the advice \mathcal{P}_{qa} . These paths aim to reflect more realistic behavior, such as not disembarking a direct train, as explained in Section 4.3. Solution quality is defined by the passenger simulation model, that includes a more detailed passenger behavior model.

5.2. Rolling Stock Rescheduling Model

The rolling stock schedule assigns rolling stock compositions to trips in the timetable, where a rolling stock composition consists of one or more units of a specific rolling stock type in a specific order. The assignment of rolling stock units to trips defines the capacity per trip in terms of the number of passengers that are able to board. At the start of the disruption, a new rolling stock plan is calculated based on a timetable \mathcal{T} that has been adjusted to the disruption, and the current, but now infeasible, rolling stock plan.

Defining \mathcal{R} as the set of all feasible rolling stock schedules, the rolling stock rescheduling problem selecting the best rolling stock schedule r can be written as

$$\min \{g(r) + f(r)\}$$

subject to

$$r \in \mathcal{R}. \quad (5)$$

Here, $g(r)$ represents the passenger-related service costs due to the occurrence of any capacity shortages, and $f(r)$ represents the operating costs of rolling stock schedule r defined by the number of composition changes, the number of changes in the shunting operations, and costs resulting from any unbalances at rolling stock depots at the end of the planning horizon. The feasible set \mathcal{R} is restricted by the maximum composition length per station, the available rolling stock at every station, and limitations in shunting opportunities and movements. Cacchiani et al. (2014) provides an overview of the state of the art in rolling stock rescheduling.

The rolling stock optimization model is, for a deterministic disruption duration, identical to Kroon, Maróti, and Nielsen (2014). The passenger simulation model (Section 4.2.2) is altered to include passenger advice. Adjustments to the model of Kroon, Maróti, and Nielsen (2014) required to deal with the uncertain disruption duration considered in this paper are discussed in Section 6.

The model of Kroon, Maróti, and Nielsen (2014) model aims to minimize both passenger inconvenience and operational cost, and to our knowledge represents the current state of the art in terms of passenger-centric rolling stock rescheduling. The model allows for an easy embedding in the solution framework in Figure 1, which is no coincidence, as this framework was inspired by Kroon, Maróti, and Nielsen (2014).

6. Uncertainty

In contrast to the common assumption in the literature, the duration of a disruption is generally unknown in practice. Operators expressed to us the concern that providing advice to passengers is not possible when the disruption duration is uncertain. Therefore, we assume that the duration of the disruption is uncertain to demonstrate that the concept of providing advice is also viable and practicable in that situation.

The uncertain disruption duration is modeled as a two stage process.

1. At the start of the disruption (denoted by τ_{start}), a set of possible disruption durations $\delta \in \mathcal{D}$ becomes available.
2. At τ_{\min} (with $\tau_{\min} > \tau_{\text{start}}$), the actual disruption duration $\delta^* \in \mathcal{D}$ is revealed.

At τ_{start} (i.e., when the disruption starts), the operator announces an estimated duration δ^{passg} and a corresponding timetable $\mathcal{T}^{\delta^{\text{passg}}}$ to the passengers who use $\mathcal{T}^{\delta^{\text{passg}}}$ to plan their journeys. The rolling stock is rescheduled assuming a duration of δ^{roll} . Parameters δ^{passg} and δ^{roll} are set independently to one of the

durations in the set \mathcal{D} , and a sensitivity analysis to these parameters is included in the case study. For the sake of convenience, we assume that τ_{\min} is equal to the minimal duration of the disruption. The proposed framework can easily be adjusted for a different choice of τ_{\min} .

The actual disruption duration δ^* , together with the corresponding timetable \mathcal{T}^{δ^*} , is revealed at τ_{\min} . At this point, the rolling stock schedule is updated, and the passengers update their paths to the shortest path in \mathcal{T}^{δ^*} .

The ARSRU algorithm is called at τ_{start} . It computes the advice and rolling stock schedules at τ_{start} by minimizing expected passenger inconvenience at τ_{start} over all possible values of $\delta^* \in \mathcal{D}$. The algorithm computes advice and rolling stock schedules until the end of the planning horizon, and takes into account that the passengers' routes and the rolling stock schedule will be updated at τ_{\min} .

Figure 3 depicts the ARSRU algorithm with uncertainty. It consists of three components: initialization at the top, the rolling stock algorithm on the left, and the advice algorithm on the right. After initialization, the rolling stock algorithm and advice algorithm iterate until a maximum number of iterations have been reached.

Input to the initialization is the set of disruptions \mathcal{D} , the set of passenger groups \mathcal{Q} , the planned timetable \mathcal{T} , and the estimated duration δ^{passg} . The initial advice a_q , the candidate advice sets \mathcal{A}_q , and the realized path sets $\mathcal{P}_{qa}^{\delta}$ are all initialized as the earliest-arriving path in the timetable $\mathcal{T}^{\delta^{\text{passg}}}$. Note that the initial advice would be equal to the passengers' preferred paths in Kroon, Maróti, and Nielsen (2014) if the actual duration δ^* were known.

The remainder of this section discusses the adjustments on the passenger simulation algorithm (Section 6.1), on the advice model (Section 6.2), and on the rolling stock rescheduling model (Section 6.3).

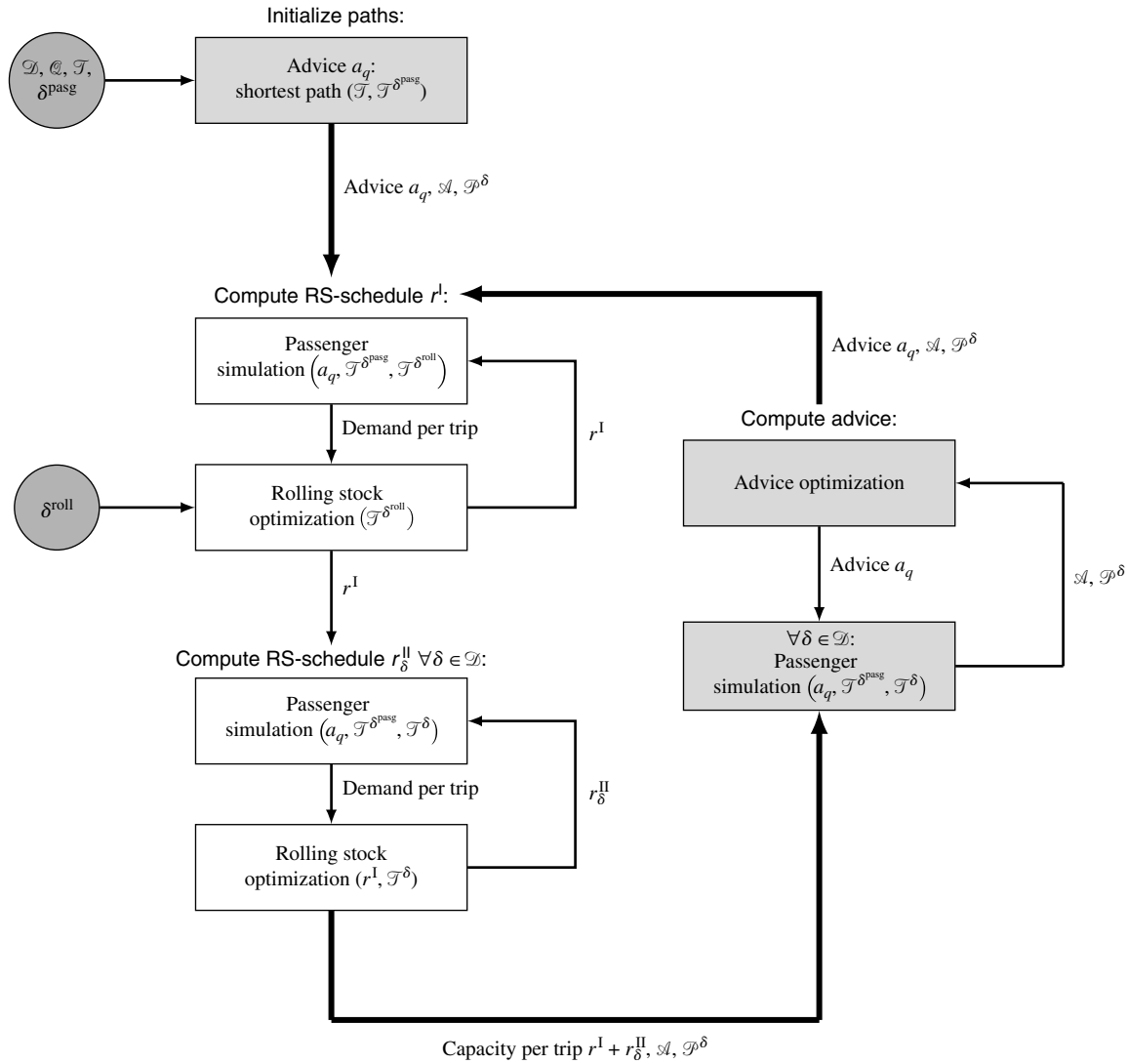
6.1. Passenger Simulation Model and Uncertainty

The simulation algorithm under uncertainty follows the same general rules as those described in Section 4. The passengers start their journeys according to the latest known timetable and eventually replan their paths at the start of the disruption (at τ_{start}) based on the announced timetable $\mathcal{T}^{\delta^{\text{passg}}}$ and on the advice a_q . Whether or not passengers accept the advice or not depends on the behavioral rules of Section 4.2.1.

The only effect of uncertainty is that the passengers learn the actual timetable \mathcal{T}^{δ^*} at τ_{\min} . As a reaction, the simulation is paused, and the passengers replan their journeys again: they all choose the earliest arriving path in \mathcal{T}^{δ^*} .

The realized paths computed in the simulation depend on the disruption duration δ^* . Consequently, the

Figure 3. ARSRU Algorithm Detailed Outline with Uncertainty



ARSRU algorithm maintains an own set \mathcal{P}_{qa}^δ of realized paths for each passenger group $q \in \mathcal{Q}$, each advice $a \in \mathcal{A}_q$, and each duration $\delta \in \mathcal{D}$. The simulation for a given δ^* adds paths to the sets $\mathcal{P}_{qa}^{\delta^*}$.

Note that the set of recommended paths for the advice \mathcal{A}_q does *not* depend on the disruption duration δ^* , as this duration is unknown at τ_{start} . Every run of the passenger simulation may provide new recommended paths to the set $a \in \mathcal{A}_q$, as described in Section 4.3.2.

We note that the scenarios give rise to graphs with different sets of arcs. Therefore, our approach treats recommended paths as *geographical paths* (i.e., as sequences of stations with direct travel possibilities between them). The simulation algorithm translates the geographical paths to actual paths (i.e., sequences of arcs) in the given timetable graph: it selects the earliest-arriving path that is consistent with the geographical path.

6.2. Advice Optimization and Uncertainty

The advice algorithm selects the advice with the overall minimal anticipated passenger inconvenience, calculated over all \mathcal{D} . After each run of the advice optimization model, the passenger simulation is performed for each $\delta \in \mathcal{D}$ in which the candidate recommended path sets \mathcal{A}_q and realized path sets \mathcal{P}_{qa}^δ are extended. The advice algorithm terminates if an iterations limit is reached or if no new paths are added to either \mathcal{A}_q or \mathcal{P}_{qa}^δ .

The advice optimization model with uncertain disruption duration is an extension of (1)–(4) by means of additional decision variables and some altered constraints. New decision variables x_{qpa}^δ represent the number of passengers who follow realized path p belonging to advice and recommended path a and passenger group q given a disruption duration of δ . The variables y_{qa} , reflecting whether or not recommended

path a is selected for passenger group q , remains independent of the disruption's duration.

Let ψ^δ denote the probability that scenario δ occurs, and c_{qp}^δ denote the inconvenience of realized path p for passenger group q given a disruption duration of δ . Finally, k_t^δ represent the capacity of trip t in adjusted timetable \mathcal{T}^δ for disruption length δ . The trip capacities are obtained from the last rolling stock scheduling step. The advice optimization model with uncertainty reads as follows:

$$\min \sum_{\delta \in \mathcal{D}} \sum_{q \in \mathcal{Q}} \sum_{a \in \mathcal{A}_q} \sum_{p \in \mathcal{P}_{qa}^\delta} \psi^\delta c_{qp}^\delta x_{qpa}^\delta$$

such that

$$\sum_{a \in \mathcal{A}_q} y_{qa} = 1 \quad \forall q \in \mathcal{Q}, \quad (6)$$

$$\sum_{p \in \mathcal{P}_{qa}^\delta} x_{qpa}^\delta = y_{qa} w_q \quad \forall q \in \mathcal{Q}, a \in \mathcal{A}_q, \forall \delta \in \mathcal{D}, \quad (7)$$

$$\sum_{q \in \mathcal{Q}} \sum_{a \in \mathcal{A}_q} \sum_{p \in \mathcal{P}_{qa}^\delta} x_{qpa}^\delta \leq k_t^\delta \quad \forall \delta \in \mathcal{D}, \forall t \in \mathcal{T}^\delta, \quad (8)$$

$$x_{qpa}^\delta \geq 0, y_{qa} \in \{0, 1\} \\ \forall \delta \in \mathcal{D}, \forall q \in \mathcal{Q}, \forall a \in \mathcal{A}_q, \forall p \in \mathcal{P}_{qa}^\delta. \quad (9)$$

The objective is changed to minimize the *expected* passenger inconvenience over all disruption durations $\delta \in \mathcal{D}$. Constraint (6) is identical to (1). Constraints (7)–(9) express that the path decomposition constraints (2), trip capacity constraints (3), and non-negativity constraints (4) hold for each duration δ .

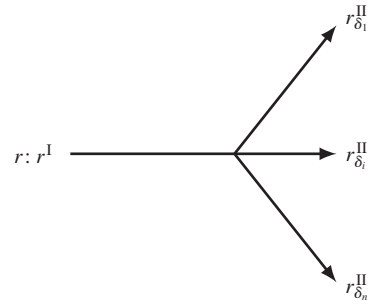
The advice optimization model with uncertain disruption duration is a two-stage stochastic program with $|\mathcal{D}|$ explicitly given scenarios. The variables y_{ap} represent the first-stage decisions, while the variables x_{qpa}^δ contain the second-stage decisions.

6.3. Rolling Stock Schedule and Uncertainty

The uncertainty of the disruption gives rise to rolling stock schedules for each possible disruption length. These schedules must coincide during the time period between τ_{start} and τ_{min} because the disruption length is revealed only at τ_{min} .

Our rolling stock algorithm under uncertainty consists of two stages. The first stage computes a new rolling stock schedule r^I for $\mathcal{T}^{\delta^{\text{roll}}}$ where δ^{roll} is the estimated duration of the disruption. The schedule r^I covers the entire planning horizon to also take into account expected off-balances at the end of the day. The second stage of the rolling stock algorithm models the rescheduling step at τ_{min} when the actual duration of the disruption is revealed. In fact, the second stage creates a rolling stock schedule r_δ^{II} for each possible disruption length $\delta \in \mathcal{D}$ since the advice optimization model relies on the trip capacities for each δ .

Figure 4. Schematic Representation of Rolling Stock Solution: A Common Part Between τ_{start} and τ_{min} is Followed by a Scenario-Dependent Part After τ_{min}



The relation between r^I and r_δ^{II} is depicted in Figure 4. The rolling stock schedules are identical during the uncertainty period between τ_{start} and τ_{min} . After τ_{min} , the schedule r^I has different continuations based on the actual disruption length δ .

The schedules r^I and r_δ^{II} are obtained by iterating between rolling stock optimization and passenger simulation as proposed by Kroon, Maróti, and Nielsen (2014). The computations for r^I and r_δ^{II} use the timetables $\mathcal{T}^{\delta^{\text{roll}}}$ and \mathcal{T}^δ , respectively. Whenever the simulation module is launched, we use the same advice; namely, the one that was computed in the last call to the advice optimization module.

The solution process starts by an initialization of passenger demand under the assumption that all trains have infinite capacity. The iterations between optimization and simulation terminate if a certain iteration limit is reached.

We note that the results of Kroon, Maróti, and Nielsen (2014) are not directly comparable to the results presented here due to the concept of uncertainty and slightly different modeling of passenger behavior.

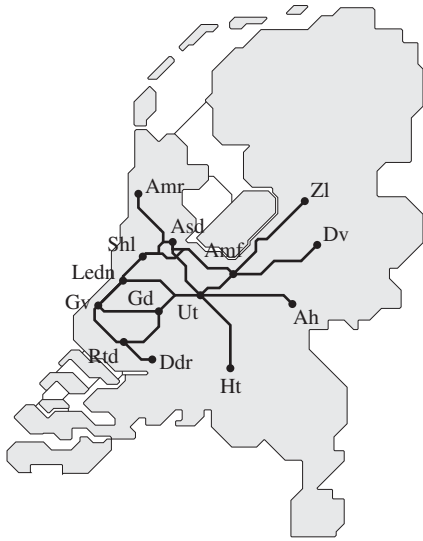
7. Computational Results

7.1. Cases and Experimental Design

The test cases concern five different disruptions in the heavily used core part of the network of Netherlands Railways, the largest passenger rail operator in the Netherlands. Figure 5 depicts the network of 14 stations connected by 938 timetable services which gives rise to 2,324 station-to-station trips. The test instances include 11,415 passenger groups, representing 422,022 passengers. The cases are derived from those in Kroon, Maróti, and Nielsen (2014), the only difference is that here we consider an uncertain duration of the disruption.

Disruptions. Each test case represents the blockage of a line segment between two stations in the network (see Table 2). The duration is three, 3.5, or four hours; these are typical values for the length of severe disruption. The half-hourly frequency of most trains motivates the

Figure 5. The Network Considered in the Test Instances



Line	Stations	Frequency
500	Gv Gd Ut Amf Zl	Hourly
700	Shl Amf Zl	Hourly
800	Amr Asd Ut Ht	Half hourly
1500	Asd Amf Dv	Half hourly
1600	Shl Amf Dv	Hourly
1700	Gv Gd Ut Amf Dv	Hourly
1900	Gv Rtd Drdr	Half hourly
2000	Gv Gd Ut Ah	Half hourly
2100	Asd Shl Ledn Gv Rtd Drdr	Half hourly
2600	Asd Shl Ledn Gv	Half hourly
2800	Rtd Gd Ut Amf	Half hourly
3000	Amr Asd Ut Ah	Half hourly
3500	Shl Ut Ht	Half hourly
8800	Ledn Ut	Half hourly
20500	Rtd Gd Ut	Hourly
21700	Rtd Gd Ut	Hourly

Table 2. Test Cases

Name	Disruption	Durations \mathcal{D}
D1	Rotterdam (Rtd)–The Hague (Gv)	3, 3.5, and 4 hours
D2	Gouda (Gd)–Utrecht (Ut)	3, 3.5, and 4 hours
D3	Utrecht (Ut)–Amersfoort (Amf)	3, 3.5, and 4 hours
D4	The Hague (Gv)–Leiden (Ledn)	3, 3.5, and 4 hours
D5	Amsterdam (Asd)–Utrecht (Ut)	3, 3.5, and 4 hours

30 minute increase per scenario. Our cases consider disruptions during the afternoon peak, as these affect the most passengers and are most likely to lead to capacity shortages. Typically, around 5,000–6,000 passenger groups are included in the advice optimization model—the others having already finished their journey. The advice optimization model contains 30,000–60,000 decision variables. Between 2% (in D1) and 17% (in D2) of the passenger groups could experience delay due to capacity shortages and therefore may receive an advice different to the shortest path. In D2 and D5, about 40% of the affected passenger groups have at least two alternatives to the shortest detour path; on average, between two and 3.4 recommended paths are available per passenger group. The ratio between realized paths and recommended paths suggests that capacity shortages are more common in these cases, at least without advice. Section 7.2.2 discusses the absolute number of passengers affected by the disruption.

Lower bounds. Both ARSRU and RSRU solutions are compared to a lower bound LB_δ , computed as the passenger inconvenience given infinite capacity and perfect information about the disruption length in scenario δ . In the lower bound, passengers follow a shortest path in the timetable adjusted to the disruption of

deterministic length. However, our cases have insufficient capacity available to accommodate all passengers on the shortest route, and the disruption length is uncertain. Thus, even optimal solutions for ARSRU cannot achieve a 0% gap; therefore, the gap does *not* represent an optimality gap. Still, the gap is a good measure for how close the solution is to an ideal solution for passengers.

We report the relative gap with the lower bound, which is defined per scenario as

$$r_\delta = \frac{PI_\delta - LB_\delta}{LB_\delta}, \quad (10)$$

where PI_δ and LB_δ denote the passenger inconvenience for the model's solution and the LB solution in scenario δ , respectively. The relative gap over all scenarios is defined as

$$r = \frac{\sum_{\delta \in \mathcal{D}} (PI_\delta - LB_\delta)}{\sum_{\delta \in \mathcal{D}} LB_\delta}. \quad (11)$$

Advice and passenger behavior. The ARSRU solution quality is influenced both by the advice and by the rolling stock schedule. To estimate the contribution of both, we evaluate five versions of our problem (see Table 3).

RSRU denotes the straightforward adaptation of the model of Kroon, Maróti, and Nielsen (2014) to include the uncertain disruption duration. In this model, there is no advice, and passengers follow the shortest path in the timetable $\mathcal{T}^{\text{pass}}$. The solutions of this model represent, to our knowledge, the state of the art in passenger-oriented rolling stock rescheduling. We will refer to these as $V0$.

V1, V2 and V3, V4 represent the rolling stock circulation computed by our ARSRU algorithm *with* the inclusion of advice, each solution *evaluated* under a different

Table 3. Overview of Problem Versions

Name	Rolling stock schedule computation	Evaluation in passenger simulation
V0	<i>RSRU</i>	$\phi = 0$
V1	<i>ARSRU</i> , $\phi = 1$	$\phi = 1$
V2	<i>ARSRU</i> , $\phi = 1$	$\phi = 0$
V3	<i>ARSRU</i> , $\phi = 1$	$\phi = \text{logit}$
V4	<i>ARSRU</i> , $\phi = 1$	$\phi = \{0, 0.2, 0.4, 0.6, 0.8, 0.95\}$

passenger behavior assumption in the passenger simulation: in V1, all passengers follow the advice; in V2, no passengers follow the advice, and passenger behavior is identical to V0; in V3, passengers follow the advice according to a logit model; and in V4, a fixed percentage of passengers follow the advice.

In the V3 and V4 evaluations, the passenger groups are split. For example, if $\phi = 0.2$, then the passenger group is split in a first group with a weight of 20% that follows the advice and a second group with a weight of 80% that follows the shortest path in $\mathcal{T}^{\delta^{\text{pass}}}$, like V0.

In the V2 evaluation, the logit model defines the probability of following the advice dependent on the relative quality of recommended path p_{qa} to the shortest path in the timetable p_t . This probability is computed as $\phi = e^{\theta c_{p_{qa}}} / (e^{\theta c_{p_{qa}}} + e^{\theta c_{p_t}})$. Parameter θ is set such that passengers are reluctant to follow an advice when the length of the advised path is (much) longer: in our experiments, when two paths differ by 10 minutes, 95% of the passengers select the shortest path p_t and only 5% follow the longer recommended path p_{qa} .

The purpose of V2 is to model passengers following their own best interests given the advice and the timetable provided to them. More realistic and advanced route choice models could be incorporated in the framework in the future. Prato (2009) presents an overview of route choice models. Parameters for such models could be estimated using survey data, such as, for example, proposed by Anderson, Nielsen, and Prato (2014) for the multimodal network of the Greater Copenhagen Area. Generally, this would require a more detailed estimation of the inconvenience of a path and possibly a distinction between different passenger types and trip purposes, as also motivated in Nielsen (2000). It is, however, outside of the scope of the current paper to develop such models.

The *ARSRU* algorithm could also compute advice and rolling stock schedules with the aforementioned different passenger behavioral models included in the simulation during computation of solutions. However, the relation between recommended paths and realized paths, resulting from only some passengers following the advice, becomes less clear in this case. Therefore, we limited ourselves to evaluating solutions of the *ARSRU* algorithm (computed assuming $\phi = 1$) under different behavior assumptions. This analysis is

Table 4. Minimum Relative Gap per Case for *RSRU* and *ARSRU*, with the *ARSRU* Solution Evaluated Under Three Passenger Behavioral Models, Computed According to (11)

Case	<i>RSRU</i> (r)	<i>ARSRU</i> (r)		
	V0	V1	V2	V3
D1	8.33	8.17	8.35	8.18
D2	35.6	16.51	31.22	26.39
D3	6.55	5.31	6.89	5.67
D4	8.86	5.98	6.68	6.20
D5	92.5	10.10	23.66	19.21

Note. The minimum is taken over all δ^{pass} and δ^{roll} .

intended as a validation of our model and as a sensitivity analysis. We leave the inclusion of different passenger behavior in the algorithm for future research.

7.2. Solution Quality

After summarizing the main results (Section 7.2.1), we discuss the different scenarios and passenger inconvenience in more detail (Section 7.2.2), as well as the performance of *ARSRU* under different behavioral models (Section 7.2.3).

7.2.1. Summary of Results. Table 4 presents the relative gap r (see (11)) of the *RSRU* and *ARSRU* solutions for the best settings of δ^{pass} and δ^{roll} per evaluation version. Per case, the relative gap of V1 always turns out to be smaller than V0, indicating that the integration of providing advice and rescheduling rolling stock can reduce passenger inconvenience during disruptions. The differences in gaps between cases indicates that some disruption locations impact passengers more severely than others. Specifically, the largest benefit of providing advice is achieved in D2 and D5, where the relative gap drops from 35.6% to 16.51% and from 92.5% to 10.1%, respectively. These cases concern the busiest, most central links in the network, for which multiple detours are available.

In the following, we aim at isolating the benefits of the different rolling stock schedule and the provision of advice in *ARSRU* separately by comparing the different versions. The online appendix contains an example that analyses the benefits of these at the microscopic level for D5.

Rolling Stock. In cases D2, D4, and D5, a better rolling stock schedule is found in *ARSRU* than in *RSRU*, as V2's relative gap is lower than V0's. In the remaining cases, D1 and D3, the *ARSRU* rolling stock schedule is very close in performance to *RSRU*—the difference is only 0.34 and 0.02 percentage points, respectively. For example, in D5, we found that the rolling stock schedule uses additional capacity on some of the popular detour routes. This reduced both the number of capacity bottlenecks at stations and the duration of the remaining bottlenecks, but increased the operational

Table 5. Summary of Solution Quality per Scenario for *RSRU* and *ARSRU*, and Percentage Difference Δ Between *RSRU* and *ARSRU*

Settings		Relative gap (r_δ)			Mean delay			Delayed passengers		
Case	Advice	S	M	L	S	M	L	S	M	L
D1	<i>RSRU</i>	11.22	6.49	7.67	34.4	33.7	34.2	12,942	14,217	15,342
D1	<i>ARSRU</i>	11.03	6.48	7.39	34.3	33.7	34.2	12,944	14,215	15,323
D1	$\Delta\%$	−2	0	−4	0	0	0	0	0	0
D2	<i>RSRU</i>	42.34	29.85	29.03	53.0	52.6	53.6	26,067	27,124	28,768
D2	<i>ARSRU</i>	18.56	14.54	16.65	50.0	51.0	53.2	22,997	24,718	26,193
D2	$\Delta\%$	−56	−51	−43	−6	−3	−1	−12	−9	−9
D3	<i>RSRU</i>	12.84	2.92	4.95	47.2	47.0	48.5	9,223	9,862	10,831
D3	<i>ARSRU</i>	11.52	2.58	2.96	48.0	46.9	47.8	8,971	9,846	10,776
D3	$\Delta\%$	−10	−12	−40	2	0	−1	−3	0	−1
D4	<i>RSRU</i>	14.16	6.36	6.88	41.9	42.3	43.3	17,486	18,358	19,611
D4	<i>ARSRU</i>	10.60	3.79	4.27	41.9	42.2	43.1	16,920	17,988	19,223
D4	$\Delta\%$	−25	−40	−38	0	0	0	−3	−2	−2
D5	<i>RSRU</i>	94.25	90.80	92.93	47.8	48.9	50.7	20,557	22,186	23,555
D5	<i>ARSRU</i>	12.25	11.24	7.30	35.4	35.4	35.3	16,058	17,862	18,784
D5	$\Delta\%$	−87	−88	−92	−26	−28	−30	−22	−19	−20

Note. r_δ is computed according to Equation (10). S, M, and L, refer to a short, medium, and long disruption length, respectively.

costs of the rolling stock model. The rolling stock optimization module (from Kroon, Maróti, and Nielsen 2014) minimizes combined operational costs (resulting, e.g., from the number of kilometers run by the rolling stock) and passenger inconvenience (such as delays). Despite the increase in operational costs, the combined objective value of the rolling stock optimization component is much lower for the *ARSRU* rolling stock circulation than in *RSRU*. We ran the *RSRU* algorithm longer, but still the best solution found was similar to Table 4 and did not come close to the *ARSRU* solution. This indicates that the rolling stock rescheduling algorithm could be improved; we leave this investigation for future research.

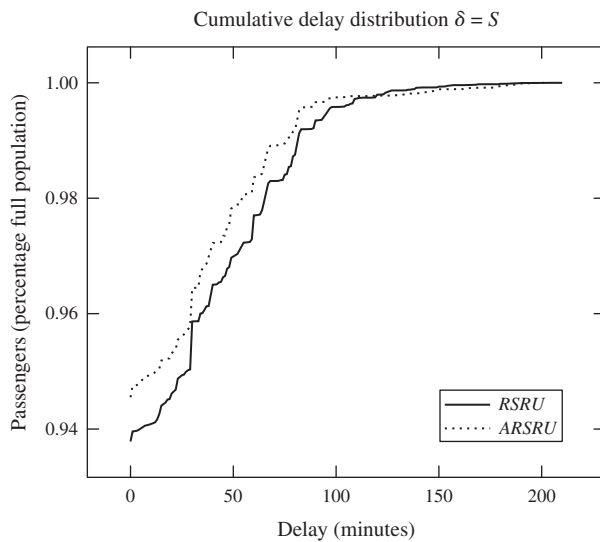
Advice. The provision of advice itself reduces passenger inconvenience further: for the same rolling stock schedule, the relative gap is lower with advice (V1) than without advice (V2) for all cases. For example, in D5, discussed in the online appendix, some passengers receive the advice for a route 10 minutes longer than the shortest path in the timetable. These passengers, however, *save* 20 minutes by avoiding the capacity bottleneck that occurs without advice in V2. This illustrates that dense networks, like that of Netherlands Railways, have viable detour options. These detour paths, when recommended, can significantly reduce passenger inconvenience.

The advised paths themselves are, in general, attractive to passengers. This is demonstrated by the small difference between full acceptance of advice (V1) and probabilistic acceptance according to a logit model (V3). In D1, D2, and D4, the relative gap difference is less than 0.36 percentage points. In D2 and D5, the difference between V1 and V3 is larger: 9.11 and 9.88 percentage points, respectively. Still, the relative gap in V3

is lower than in V2 where no passengers follow the advice. Moreover, we set the logit model to be very conservative; hence, if passengers were slightly willing to make detours (of, say, 10 minutes, like in the advice of D5), one could expect even better results. Overall, V3 gaps are lower than V0 and V2, thus indicating that the advice is beneficial to passengers. In particular, our algorithm finds candidate recommended paths that are generally in the best interest of the passengers.

7.2.2. Scenarios and Passenger Inconvenience. This section discusses results per scenario (short, medium, long) and the passenger delay distribution. Table 5 provides a closer look at the relative gap r_δ (see (10)) per disruption duration, as well as at the absolute mean delay per affected passenger, and at the total number of delayed passengers. The latter two include unavoidable passenger inconvenience even with perfect information and unlimited capacity; therefore, the change in relative gap r_δ is larger than the change in absolute delay per passenger and number of affected passengers. Rows $\Delta\%$ show the relative difference between *RSRU* and *ARSRU*.

The *ARSRU* solutions reduce the relative gap r_δ for all disruption lengths, mostly by reducing the number of affected passengers—for example, by 10% in D2 and by 20% in D5. Indeed, the *ARSRU* solutions for D2 and D5 have much fewer passengers who are not able to board a train. Moreover, the average delay for all affected passengers is reduced in most cases; case D5 has a reduction of up to 30%. The small improvements in D1, D3, and D4 are in line with previously found small improvements in Kroon, Maróti, and Nielsen (2014) and may be an indication that solutions are close

Figure 6. Cumulative Delay Distribution for Case D2, $\delta = S$ 

to optimal. The development of better lower bounds in future research could validate this hypothesis.

Figure 6 depicts the cumulative delay distributions for D2. The horizontal axis shows the delay minutes, and the vertical axis indicates the percentage of passengers that experienced at most a given amount of delay. The ARSRU solution for D2 reduces the number of delayed passengers by 13%, from 5.4% to 6.2% of the population. ARSRU's cumulative delay distribution graph is above RSRU in most of Figure 6; this indicates that most of the passengers (96% of delayed passengers) experience smaller delays in ARSRU than in RSRU. The other test cases behave similarly but are also able to reduce all worst case delays. Case D5 is most pronounced, where 19% less passengers are affected by the disruption. The number of affected passengers drops from 4.7% to 3.8% of the full population.

7.2.3. Passenger Behavior and Parameters. Table 6 contains the relative gap r (see (11)) for ARSRU and RSRU solutions for disruption cases D1–D5 over all settings of δ^{pass} and δ^{roll} , and for ARSRU for the different behavioral models. Each row from left to right presents, respectively, the relative gap given a logit model (V3), full compliance (V1), fixed compliance of advice (from 95% to 20% of passengers as V4), no advice (V2, 0% compliance), and finally the relative gap in the RSRU solution with no advice (V0). The minimum over all possible settings of δ^{pass} and δ^{roll} for versions V0–V3 were presented in Table 4 and are also marked in bold in Table 6.

Both δ^{pass} and δ^{roll} have, in general, a notable influence on the quality of the solution. Defining the variance as the difference between the minimum and maximum relative gap per case, RSRU has a variance between 3 and 21 percentage points per case. ARSRU's variance is commonly smaller and between 0.34 and

15.17 percentage points; only for D2 (13.9 percentage points) is it higher than RSRU (7.88). For both ARSRU and RSRU, the variance is lowest for D1 and highest for D5. The variance increases when less passengers follow the advice. The best settings for V2 and V0 (both no advice) generally seem to coincide, and $\delta^{\text{pass}} = M$, $\delta^{\text{roll}} = M$ give the best or close to the best results. ARSRU V1 results would improve over RSRU given these settings for all cases but in D1, D2, and D5 yields better results with $\delta^{\text{pass}} = S$.

The best selection of δ^{pass} and δ^{roll} also depends on the behavioral model. In general, a long δ^{pass} gives better results when not all passengers follow the advice. If $\delta^{\text{pass}} = S$, more passengers than on an undisrupted day will be waiting for this first train, as soon as waiting is faster than starting a detour. In the case of $\delta^* = S$, this train will depart but may not have sufficient capacity for all waiting passengers, causing additional delays for passengers unable to board. In the case of $\delta^* = M$ or $\delta^* = L$, this train will not depart. Passengers who have lost time waiting in vain for the first train will likely start a detour after all, and will face a surge of demand on the detour route—consisting of passengers waiting for the first train and passengers that were planning to take this train in an undisrupted situation. This again may lead to capacity bottlenecks. Therefore, more passenger delays arise in general if the end of the disruption is announced too far in advance, or if its duration is underestimated. Future research could focus on the influence of the timing of the disruption's definite end announcement. Moreover, the dependence on δ^{roll} indicates that a further development of robust rolling stock rescheduling measures may be a second way to improve passenger service during disruptions.

7.3. Computation Time and Convergence

All computational experiments were run using CPLEX version 12.6 on an Intel I7-4800MQ 2.7 GHz processor. The algorithm is implemented in Java.

7.3.1. Computation Time. The computation times for the full model and the individual components in Table 7 are in minutes per case, allowing for two iterations between the algorithms for advice optimization and rolling stock optimization, and two iterations between the passenger simulation model and the advice optimization model or the rolling stock rescheduling model. One iteration of the rolling stock algorithm contains $|\mathcal{D}| + 1$ calls to the rolling stock optimization model. One iteration of the advice algorithm calls the advice optimization model once and the passenger simulation $|\mathcal{D}|$ times. More information can be found in Section 6. Additional computational experiments showed that more iterations only led to minor improvements in solution quality: a maximum of 2.2 percentage points in r for D2, 1.3 percentage points in r for D5, and less than 0.2 percentage points in r for the

Table 6. Overview of Relative Gap r (Computed According to Equation (11)) for $RSRU$ and $ARSRU$ for Different Behavioral Settings

Case	Settings		ARSRU (r)								RSRU (r)
			V3	V1	Compliance rate (V4)					V2	
	δ^{passg}	δ^{roll}	P	1.0	0.95	0.8	0.6	0.4	0.2	0.0	V0
D1	S	S	14.24	8.51	8.69	9.35	10.44	11.75	13.59	15.42	10.50
D1	S	M	13.90	8.17	8.19	8.49	9.62	11.11	13.18	15.30	10.91
D1	S	L	14.33	8.17	8.19	8.57	9.83	11.53	13.63	15.68	11.38
D1	M	S	8.65	8.51	8.52	8.54	8.57	8.60	8.63	8.67	8.65
D1	M	M	8.23	8.17	8.18	8.20	8.24	8.27	8.31	8.35	8.33
D1	M	L	8.23	8.17	8.18	8.20	8.24	8.27	8.31	8.35	8.33
D1	L	S	8.71	8.51	8.55	8.67	8.82	8.98	9.13	9.29	9.29
D1	L	M	8.18	8.17	8.22	8.34	8.51	8.67	8.84	9.01	9.06
D1	L	L	8.18	8.17	8.22	8.34	8.51	8.67	8.84	9.01	9.06
D2	S	S	27.75	16.56	17.13	19.51	22.60	25.63	28.80	32.18	42.11
D2	S	M	26.75	16.51	16.70	18.18	21.00	24.34	27.72	31.22	39.79
D2	S	L	36.82	25.95	26.40	28.06	30.81	33.93	37.16	40.53	43.45
D2	M	S	33.74	29.96	30.14	31.00	32.31	33.65	34.99	36.33	36.48
D2	M	M	29.52	24.53	24.85	26.02	27.55	29.11	30.64	32.24	36.01
D2	M	L	29.96	26.90	27.03	27.57	28.54	29.68	30.91	32.30	37.59
D2	L	S	34.88	30.41	30.49	31.85	33.59	35.23	36.95	38.75	38.26
D2	L	M	26.39	23.58	23.93	25.05	26.68	28.42	30.23	32.18	36.16
D2	L	L	36.22	30.27	30.78	32.45	34.82	37.14	39.54	42.26	35.57
D3	S	S	14.42	6.10	6.38	7.31	9.06	11.20	13.61	16.29	12.60
D3	S	M	12.88	5.31	5.48	6.33	7.87	9.75	12.10	14.74	10.77
D3	S	L	14.11	6.15	6.46	7.51	9.07	10.91	13.25	16.14	13.10
D3	M	S	6.95	6.10	6.18	6.42	6.74	7.09	7.53	8.03	6.62
D3	M	M	5.96	5.31	5.37	5.56	5.81	6.10	6.50	6.89	6.55
D3	M	L	7.08	6.18	6.25	6.48	6.79	7.16	7.60	8.10	7.11
D3	L	S	6.38	6.11	6.16	6.32	6.54	6.81	7.08	7.34	7.11
D3	L	M	5.67	5.46	5.53	5.74	6.01	6.30	6.67	7.08	6.84
D3	L	L	6.52	6.26	6.30	6.44	6.63	6.87	7.11	7.34	7.11
D4	S	S	15.32	8.22	8.38	8.88	9.92	11.56	13.79	16.30	19.26
D4	S	M	13.68	6.04	6.22	6.95	8.20	9.87	12.15	14.68	19.14
D4	S	L	15.09	6.05	6.23	7.15	8.82	10.74	13.25	16.19	20.05
D4	M	S	6.42	6.04	6.05	6.10	6.17	6.23	6.42	6.68	8.86
D4	M	M	6.41	5.98	6.00	6.06	6.14	6.21	6.41	6.69	8.87
D4	M	L	8.67	8.25	8.27	8.33	8.40	8.48	8.65	8.86	8.87
D4	L	S	6.26	6.07	6.13	6.32	6.57	6.82	7.20	7.65	9.83
D4	L	M	6.20	5.99	6.05	6.26	6.52	6.79	7.18	7.65	9.83
D4	L	L	8.50	8.27	8.34	8.54	8.80	9.07	9.46	9.93	9.83
D5	S	S	21.95	10.10	10.32	11.33	13.08	17.82	24.75	36.44	103.16
D5	S	M	26.67	11.88	12.05	12.82	15.49	20.60	27.35	37.92	113.53
D5	S	L	40.63	12.84	13.41	15.41	20.83	31.05	41.50	51.89	108.33
D5	M	S	19.21	11.02	11.07	11.35	12.61	15.63	19.77	25.24	93.98
D5	M	M	20.71	12.38	12.38	12.59	14.11	16.70	21.10	27.34	94.31
D5	M	L	38.33	25.27	26.01	28.33	31.38	34.73	38.54	42.62	98.97
D5	L	S	18.38	11.02	11.01	11.10	12.11	14.83	18.58	23.66	92.53
D5	L	M	20.18	12.38	12.34	12.43	13.79	16.22	20.47	26.56	92.61
D5	L	L	37.77	25.27	25.96	28.12	31.02	34.14	37.62	41.33	92.99

Note. Bold are best solutions over settings δ^{passg} and δ^{roll} for solutions where passenger behavior follows a logit model, follows the advice, or never follows the advice (left to right), and for solutions $RSRU$.

other cases. Table 7 provides the mean and maximum computation times per case over all possible settings for δ^{passg} and δ^{roll} .

The optimization model solves in less than a minute for the advice optimization and a few seconds for the rolling stock optimization. The passenger simulation runs between 1.5 and 10 seconds, most commonly in two seconds or less. An iteration of the advice optimization requires between 0.49 and 2.33 minutes.

Longer computation times were associated with a bigger positive impact of advice on the solution quality. A rolling stock iteration requires between one and two minutes.

The total computation time is between 4.33 and 11.59 minutes. There are opportunities to decrease computation time by a more efficient implementation, for instance, using parallel computation in (part of) the algorithm. Specifically, r_{δ}^{II} is currently computed

Table 7. Computation Times (in Minutes) for the Full Algorithm, Solving the Mathematical Optimization Models, a Single Iteration of the Subalgorithms, and the Passenger Simulation Model

Case	Rolling stock						Advice				Passenger	
	Full		Optimization		Algorithm It.		Optimization		Algorithm It.		Simulation	
	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max
D1	4.33	4.45	0.03	0.17	1.03	1.11	0.24	0.25	0.49	0.69	0.03	0.03
D2	10.55	11.59	0.03	0.07	2.01	2.08	0.69	0.8	2.11	2.33	0.11	0.18
D3	4.96	5.15	0.03	0.09	1.09	1.11	0.3	0.31	0.71	0.79	0.03	0.03
D4	5.05	5.22	0.03	0.11	1.11	1.24	0.28	0.29	0.75	0.76	0.03	0.03
D5	7.58	8.45	0.03	0.11	1.65	1.77	0.7	0.85	1.65	1.93	0.05	0.17

sequentially for all scenarios but could be computed in parallel as they are independent. Similarly, the passenger simulation could be run in parallel for different scenarios in the advice optimization algorithm, with similar savings. The individual components of the mathematical optimization model and simulations are relatively fast (Table 7); therefore, limited benefit is expected from using parallelization within these components.

7.3.2. Convergence. The *ARSRU* algorithm is not guaranteed to converge. This is in line with the observations of Kroon, Maróti, and Nielsen (2014) on the nonconverging character of their rolling stock optimization framework. In this section, we illustrate the typical behavior of *ARSRU* by studying how the relative gap progresses. We solve disruption cases D1–D5. In these runs, we allow five iterations of the inner loop for rolling stock and *at most* five iterations of the inner loop for advice. We terminate the inner loop for advice if no new candidates or realized paths are found, and return the best result found so far. Finally, the outer loop is limited to five iterations.

Figure 7 gives the progression of the relative gaps r_g (see (10)) for each disruption length and the best settings of δ^{pass} and δ^{roll} . The vertical axis represents the relative gap, and the horizontal axis shows the number of calls to the advice optimization model. The markers on the lines indicate the data points right after finishing an inner loop for rolling stock. That is, the markers delineate the outer loop iterations. The sharp vertical changes, such as those in D2, D4, and D5, correspond to an improvement thanks to a superior rolling stock solution.

The lines in Figure 7 follow two patterns: either no improvement on the initial solution or a two-step reduction where the first reduction arises from advice optimization and the second reduction is achieved in the subsequent call to the rolling stock module. Note that, in most cases, the best solution is found in the first outer iterations and within the first two inner iterations. D2 behaves atypically: it has an improvement of 2 percentage points in the third outer iteration.

While different settings of δ^{pass} and δ^{roll} mostly give lines very similar to those in Figure 7, a few rare cases show other interesting patterns. The left diagram in Figure 8 indicates a case where advice optimization iterations yield successively worse solutions but the subsequent rolling stock optimization step recovers solution quality. The right diagram in Figure 8 is an example for the opposite case where an inferior solution of rolling stock optimization is improved by subsequent advice optimization runs.

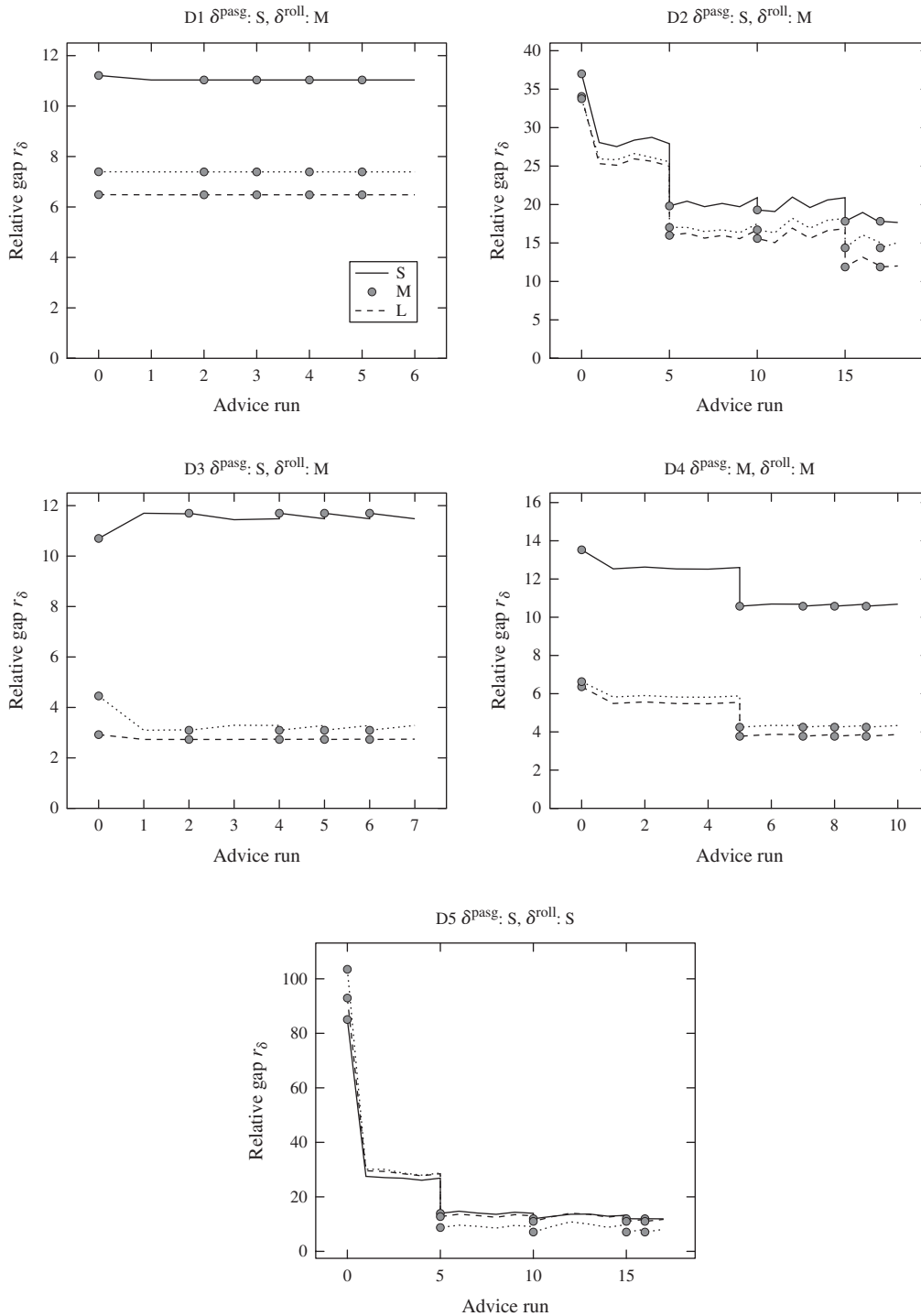
In summary, the convergence of the *ARSRU* algorithm is not guaranteed, and the settings of δ^{pass} and δ^{roll} have a strong influence on the convergence. In most test cases and for most settings, the best solution is found within the first two iterations, which motivated our selection of two iterations for the experiments. Adding more iterations is rarely beneficial, and it may occasionally even worsen the solution.

8. Conclusion and Discussion

This paper presents an optimization algorithm for improving passenger service during major disruptions with an uncertain duration. It provides personalized route advice supported by rolling stock rescheduling. The algorithm includes more realistic dynamic passenger behavior in systems with free route choice than an operator control or flow cost minimization model. Route advice is intended to be in the best interest of the passengers receiving it. We evaluate solutions under various assumptions on passenger behavior, including where passengers ignore the advice. We compare our approach to the state-of-the-art passenger-oriented rescheduling framework (Kroon, Maróti, and Nielsen 2014) that does *not* use advice.

Results on realistic test cases indicate that the integrated advice and rolling stock scheduling solutions of the proposed *ARSRU* algorithm significantly reduces the passenger inconvenience, even when the duration of the disruption is uncertain and when not all passengers follow the advice. The number of passengers affected by the disruption is reduced by up to 20%, and the average delay is reduced by up to 30%. The model

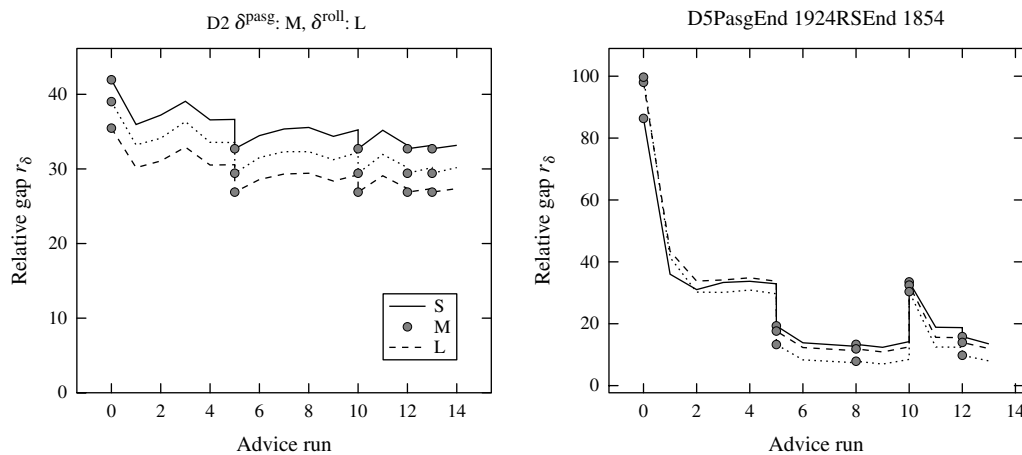
Figure 7. Convergence of Cases D1–D5 for Best Settings of δ^{passg} and δ^{roll}



is successful in advising routes that are attractive to individual passengers; indeed, the solution quality is still good when passengers only accept advice according to a logit model, where the recommended route in the advice is weighed against the expected shortest path in the timetable. In addition, the solution quality only diminishes slowly when increasing the number of passengers who do not follow the advice.

We compare the *ARSRU* algorithm to an existing passenger-oriented rescheduling model that does not consider travel advice. Our computational tests indicate that, in three out of five cases, service quality benefits from travel advice even if some passengers decide to disregard it. In some test cases, we experience a dramatic improvement even if all passengers ignore the advice. We believe that the improvement is because

Figure 8. Two Additional Patterns That Occurred: Increase in Passenger Inconvenience Over Multiple Advice Loops, and Increase in Passenger Inconvenience Due to New Rolling Stock Schedule



of a more refined prediction of the likely passengers flows. Better anticipated demand leads to better rolling stock schedules that are able to prevent the most severe capacity bottlenecks. An investigation of these details is the subject of our ongoing research.

Areas for future research include extending the passenger behavioral model, considering different ways of modeling the uncertain disruption length, developing a robust rolling stock rescheduling module, and efforts to increase the speed of the algorithm. In particular, we wonder how our algorithm behaves if passengers receive individually customized travel advice rather than the same advice as each member of a passenger group. Moreover, future research could use the current wealth of data on passenger demand to fine-tune, validate, and possibly add to, the behavioral assumptions in the simulation and solution approach.

Acknowledgments

The authors thank Professor Peter Vervest for his support on pursuing this topic. This paper is dedicated to the dear memory of Leo Kroon who passed away unexpectedly on the 14th of September, 2016. The other authors want to express their gratitude for all of the inspiration, support, patience, and friendship. Farewell, Leo.

References

- Anderson MK, Nielsen OA, Prato CG (2014) Multimodal route choice models of public transport passengers in the greater Copenhagen area. *Eur. J. Transportation Logist.* 6(3):1–25.
- Barnhart C, Belobaba P, Odoni AR (2003) Applications of operations research in the air transport industry. *Transportation Sci.* 37(4):368–391.
- Bratu S, Barnhart C (2006) Flight operations recovery: New approaches considering passenger recovery. *J. Scheduling* 9(3): 279–298.
- Cacchiani V, Caprara A, Galli A, Kroon LG, Maroti G, Toth P (2012) Railway rolling stock planning: Robustness against large disruptions. *Transportation Sci.* 46(2):217–232.
- Cacchiani V, Huisman D, Kidd MP, Kroon LG, Toth P, Veelenturf LP, Wagenaar J (2014) An overview of recovery models and algorithms for real-time railway rescheduling. *Transportation Res. Part B: Methodological* 63:15–37.
- Cadarso L, Marín A (2011) Robust rolling stock in rapid transit networks. *Comput. Oper. Res.* 38(8):1131–1142.
- Cadarso L, Marín A, Maróti G (2013) Recovery of disruptions in rapid transit networks. *Transportation Res. Part E: Logist. Transportation Rev.* 53:15–33.
- Cadarso L, Maróti G, Marín A (2015) Smooth and controlled recovery planning of disruptions in rapid transit networks. *IEEE Trans. Intelligent Transportation Systems* 16(4):2192–2202.
- Ceder A, Wilson NHM (1986) Bus network design. *Transportation Res. Part B: Methodological* 20(2):331–344.
- Corman F, D'Ariano A, Marra AD, Pacciarelli D, Samá M (2017) Integrating train scheduling and delay management in real-time railway traffic control. *Transportation Res. Part E: Logist. Transportation Rev.* 105:213–239.
- Dauzère-Pérés S, De Almeida D, Guyon O, Benhizia F (2015) A Lagrangian heuristic framework for a real-life integrated planning problem of railway transportation resources. *Transportation Res. Part B: Methodological* 74:138–150.
- Dollevet T, Huisman D, Schmidt M, Schöbel A (2012) Delay management with rerouting of passengers. *Transportation Sci.* 46(2):74–89.
- Fioole P, Kroon L, Maroti G, Schrijver A (2006) A rolling stock circulation model for combining and splitting of passenger trains. *Eur. J. Oper. Res.* 174(2):1281–1297.
- Haahr JT, Pisinger D, Larsen J (2015) Reactive robustness and integrated approaches for railway optimization problems. Unpublished doctoral thesis, DTU Management Engineering, Technical University of Denmark, Lyngby, Denmark.
- Haahr JT, Wagenaar J, Veelenturf LP, Kroon LG (2016) A comparison of two exact methods for passenger railway rolling stock (re)scheduling. *Transportation Res. Part E: Logist. Transportation Rev.* 91:15–32.
- Hu Y, Song Y, Zhao K, Xu B (2016) Integrated recovery of aircraft and passengers after airline operation disruption based on a {GRASP} algorithm. *Transportation Res. Part E: Logist. Transportation Rev.* 87:97–112.
- Hu Y, Xu B, Bard JF, Chi H, Gao M (2015) Optimization of multi-fleet aircraft routing considering passenger transiting under airline disruption. *Comput. Indust. Engrg.* 80:132–144.
- Jespersen-Groth J, Potthoff D, Clausen J, Huisman D, Kroon LG, Maróti G, Nyhave Nielsen M (2009) Disruption management in passenger railway transportation. Ahuja RK, Möhring RH, Zorilagiis C, eds. *Robust and Online Large-Scale Optimization—Models and Techniques for Transportation Systems*, Lecture Notes Comput. Sci., Vol. 5868 (Springer, Berlin Heidelberg), 399–421.

- Kohl N, Larsen A, Larsen J, Ross A, Tiourine S (2007) Airline disruption management: Perspectives, experiences and outlook. *J. Air Transport Management* 13(3):149–162.
- Koutsopoulos HN, Cats O, Toledo T, Burghout W (2011) Effect of real-time transit information on dynamic path choice of passengers. *Transportation Res. Record* 2217:46–54.
- Kroon LG, Maróti G, Nielsen LK (2014) Rescheduling of railway rolling stock with dynamic passenger flows. *Transportation Sci.* 49(2):165–184.
- Lan S, Clarke J, Barnhart C (2006) Planning for robust airline operations: Optimizing aircraft routings and flight departure times to minimize passenger disruptions. *Transportation Sci.* 40(2):15–28.
- Maher SJ (2016) Solving the integrated airline recovery problem using column-and-row generation. *Transportation Sci.* 50(1):216–239.
- Nielsen LK, Kroon L, Maróti G (2012) A rolling horizon approach for disruption management of railway rolling stock. *Eur. J. Oper. Res.* 220(2):496–509.
- Nielsen OA (2000) A stochastic transit assignment model considering differences in passengers utility functions. *Transportation Res. Part B: Methodological* 34(5):377–402.
- Parbo J, Nielsen OA, Prato CG (2014) User perspectives in public transport timetable optimisation. *Transportation Res. Part C: Emerging Tech.* 48:269–284.
- Parbo J, Nielsen OA, Prato CG (2015) Passenger perspectives in railway timetabling: A literature review. *Transportation Rev.* 36(4):500–526.
- Prato CG (2009) Route choice modeling: Past, present and future research directions. *J. Choice Modelling* 2(1):65–100.
- Qu W, Corman F, Lodewijks G (2015) A review of real time railway traffic management during disturbances. Corman F, Voß S, Negenborn RR, eds. *Computational Logistics*, Lecture Notes Comput. Sci., Vol. 9335 (Springer International Publishing, Cham, Switzerland), 658–672.
- Samá M, D'Ariano A, Corman F, Pacciarelli D (2016) A variable neighbourhood search for fast train scheduling and routing during disturbed railway traffic situations. *Comput. Oper. Res.* 78:480–499.
- Sato K, Tamura K, Tomii N (2013) A mip-based timetable rescheduling formulation and algorithm minimizing further inconvenience to passengers. *J. Rail Transport Planning Management* 3(3):38–53.
- Schöbel A (2007) Integer programming approaches for solving the delay management problem. Geraets F, Kroon LG, Schöbel A, Wagner D, Zaroliagis CD, eds. *Algorithmic Methods for Railway Optimization*, Lecture Notes Comput. Sci., Vol. 4359 (Springer, Berlin Heidelberg), 145–170.
- van der Hurk E, Koutsopoulos HN, Wilson N, Kroon LG, Maróti G (2016) Shuttle planning for link closures in urban public transport networks. *Transportation Sci.* 50(3):947–965.
- Veelenturf LP, Kroon LG, Maróti G (2017) Passenger oriented railway disruption management by adapting timetables and rolling stock schedules. *Transportation Res. Part C: Emerging Tech.* 80(July):133–147.
- Veelenturf LP, Potthoff D, Huisman D, Kroon LG, Maróti G, Wagelmans APM (2016) A quasi-robust optimization approach for crew rescheduling. *Transportation Sci.* 50(1):204–215.
- Watkins KE, Ferris B, Borning A, Rutherford GS, Layton D (2011) Where is my bus? Impact of mobile real-time information on the perceived and actual wait time of transit riders. *Transportation Res. Part A: Policy Practice* 45(8):839–848.